PROBABILITY

Probability is the numerical measure of the likelihood of the required outcome happening or not.
It is stated as a fraction or as a ratio of the number of required outcomes to the number of possible outcomes

$$\text{Probability of an event} = \frac{\text{No. of required outcomes}}{\text{No. of possible outcomes}}$$

Probability of a certainity is 1 and the probability of an impossibility is 0.
There fore probability lies between 0 and 1.
ie $0 \leq P(A) \leq 1$

The sample space of tossing a coin is a Head (H) and a tail (T).
Below are sample spaces of the following:-

(i) tossing 2 coins

Since the sample space of a coin is H T
For the 2 coins

		2nd coin H	T
1st coin	H	HH	HT
	T	TH	TT

There fore the sample space for tossing 2 coins is
HH
HT
TH
TT

a) Find the Probability of getting a head.
P (Head) = 3

b) Probability of getting 2 heads

$P(2 \text{ heads}) = \frac{2}{4} = \frac{1}{2}$

(ii) tossing 3 coins

Consider the sample space of the 2 coins and for the third coin.

	3rd coin H	T
Two coins		
HH	HHH	HHT
HT	HTH	HTT
TH	THH	THT
TT	TTH	TTT

Therefore the sample space of tossing 3 coins is

HHH	HHT
HTH	HTT
THH	THT
TTH	TTT

Find

a) Probability of 2 heads and a tail

$P(2 \text{ Heads and a tail}) = \frac{3}{8}$

(iii) Sample space of tossing a coin and throwing a die

A die has 6 faces ~~labled~~ labeled 1, 2, 3, 4, 5, 6

∴ Sample space of a die is 1 2 3 4 5 6

If we combine the sample space of a coin and that

Die	1	2	3	4	5	6
Coin H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

we have 12 possibilities

∴ Probability of getting a tail and a Prime number will be given as

$$P(T \text{ and a prime number}) = \frac{3}{12}$$
$$= \frac{1}{4}$$

a) Find the Probability of getting a Head and a Square number.

$$P(H \text{ and a Square number}) = \frac{2}{12}$$
$$= \frac{1}{6}$$

b) The Probability of getting a Head and an odd number

$$= P(H \text{ and Odd number}) = \frac{3}{12}$$
$$= \frac{1}{4}$$

IV) Sample Space of throwing 2 dice

Consider the sample spaces for each die and combine them.

1st Die \ 2nd Die	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

The number of possibilities is 36

a) Find the Probability that ~~th~~

(i) the Sum of the numbers shown on the dice is a Prime number.

$P(\text{sum is a prime number}) = \frac{15}{36} = \frac{5}{12}$

(ii) ~~P(sum is an odd~~ the sum is an odd number

$P(\text{sum is an odd number}) = \frac{18}{36}$

$= \frac{1}{2}$

(iii) the sum is a factor of 6

$P(\text{sum is a factor of } 6) = \frac{8}{36}$

$= \frac{2}{9}$

Types of Probability

There are two types of probability namely:-

(i) Experimental probability
(ii) Theoretical probability

Experimental Probability

This is the probability based on experimental records ie Numerical records of the past are used to pedict the future.

For example A girl is interested in predicting her 1st born and below is the record of children in her family.

	Male	Female
Mother's sister	6	8
Mother's brother	4	8
Father's sister	5	8
Father's brother	2	5
Mother	7	7
Total	24	36

a) Find the experimental probability that the girl's first born is female.

Total no of females is 36

$\therefore P(\text{First born is female}) = \frac{36}{60}$

$= \frac{3}{5}$

b) If the girl eventually has 10 children, how many are most likely to be boys?

$P(\text{getting a boy}) = \frac{24}{60} = \frac{2}{5}$

$\therefore$ The number of boys $= \frac{2}{\cancel{5}_1} \times \cancel{10}^2$

$= 4$ boys

NOTE : If the probability of an event A is P(A) then the probability of event A not happening will be P(A').

$\therefore$ The $P(A') = 1 - P(A)$

Eg: If the probability of passing an exam is $2/3$, what is the probability of failing?

$P(\text{Failing}) = 1 - 2/3$

$= 1/3$

THEORETICAL PROBABILITY

This is probability based on the physical nature of a given situation being constant eg

A card is picked at random from a pack of 52 playing cards. What is the probability that it is a King?

There are 4 Kings in the park of cards therefore

$P(\text{picking a King}) = \frac{4}{52}$

$= \frac{1}{13}$

MUTUALLY EXCLUSIVE EVENTS

Two events are mutually exclusive if either of them can occur but not both. This means these events have no intersection eg
A set of vowels and a set of Prime numbers.

LAWS OF PROBABILITY

1. Additional law of probability.
This law states that given 2 events A and B then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

But if the events are mutually exclusive,
$P(A \cup B) = P(A) + P(B)$.
NB $P(A \cup B)$ means Probability of A or B occuring.
Example.
1 A number is chosen at random from the even numbers from 2 - 20. Find the probability that the number is a factor of 18 or a multiple of 5

Sample Space = $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$F_{18} = \{2, 6, 18\}$

$M_5 = \{10, 20\}$ But $P(F_{18} \cap M_5) = \emptyset$

$$\therefore P(F_{18} \cup M_5) = P(F_{18}) + P(M_5)$$

$$= \frac{3}{10} + \frac{2}{10}$$

$$= \frac{5}{10}$$

$$= \frac{1}{2}$$

2. Given that A represents numbers 1 to 15 and a number is chosen at random from A, Find the probability that the number is a multiple of 2 or 5.

The sample space is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$M_2 = \{2, 4, 6, 8, 10, 12, 14\}$

$M_5 = \{5, 10, 15\}$

$M_2 \cap M_5 = \{10\}$

$$P(M_2 \cup M_5) = P(M_2) + P(M_5) - P(M_2 \cap M_5)$$

$$= \frac{7}{15} + \frac{3}{15} - \frac{1}{15}$$

$$= \frac{9}{15}$$

$$= \frac{3}{5}$$

Exercise

1. What is the probability that ~~the~~ a number chosen at random from a set of the first 15 numbers is an odd number or a number divisible by 3?

2. The numbers 3, 4 & 5 are arranged in a random manner to form a three digit number. No digit is repeated in the number formed.

(i) Write down the sample space

(ii) Find the probability that the number formed is even.

3. A box contains red, white and black balls. The probability of picking a red ball is $2/5$ and that of a white ball is $1/6$. What is the probability of picking a black ball?

INDEPENDENT EVENTS

Two events A and B are independent if the outcome of A is not related to the outcome of B. Therefore if A and B are independent events, the probability of A and B occurring is the product of their individual probabilities

ie $P(A \cap B) = P(A) \times P(B)$

NB $P(A \cap B)$ means the Probability of A and B occuring. This is the multiplication law of probability.

Example.

A die is ~~thrown~~ thrown and a coin is tossed. What is the probability of getting a head and No. 3?

From the previous sample space we had

Coin	Die 1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

$\therefore P(H \cap No.3) = \frac{1}{12}$

Now using the multiplication law of probability

$P(H) = \frac{1}{2}$ and $P(3) = \frac{1}{6}$

$\therefore P(H \cap No.3) = P(H) \times P(3)$

$= \frac{1}{2} \times \frac{1}{6}$

$= 1/12$

Eg2. A bag contain's 3 black balls and 2 white balls. A ball is taken from the bag and then replaced and the second ball is chosen.

What is the probability that;

(i) Both balls are black?

$$P(B \cap B) = P(B) \times P(B)$$

$$= \frac{3}{5} \times \frac{3}{5}$$

$$= \frac{9}{25}$$

(ii) Both balls are white?

$$P(W \cap W) = P(W) \times P(W)$$

$$= \frac{2}{5} \times \frac{2}{5}$$

$$= \frac{4}{25}$$

(iii) P(white and then Black)

The possibilities are $P(W \cap B)$ or $P(B \cap W)$

$$= P(W \cap B) + P(B \cap W)$$

$$= P(W) \times P(B) + P(B) \times P(W)$$

$$= \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5}$$

$$= \frac{6}{25} + \frac{6}{25}$$

$$= \frac{12}{25}$$

Now if the 2 balls are picked from the bag without replacement,

(i) $P(B \cap B) = P(B) \times P(B)$

$= \frac{3}{5} \times \frac{2}{4}$

$= \frac{6}{20}$

$= \frac{3}{10}$

ie If we pick the black ball the first time, then $P(B) = \frac{3}{5}$. If we don't put the ball back in the bag, the number of black balls becomes 2 and the total number of balls in the bag is now 4. ~~So the P(B)~~ So after the first pick, the $P(B)$ is now $\frac{2}{4}$.

(ii) $P(W \cap W) = P(W) \times P(W)$

$= \frac{2}{5} \times \frac{1}{4}$

$= \frac{1}{10}$

(iii) $P(W \cap B) + P(B \cap W)$

$= P(W) \times P(B) + P(B) \times P(W)$

$= \frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{2}{4}$

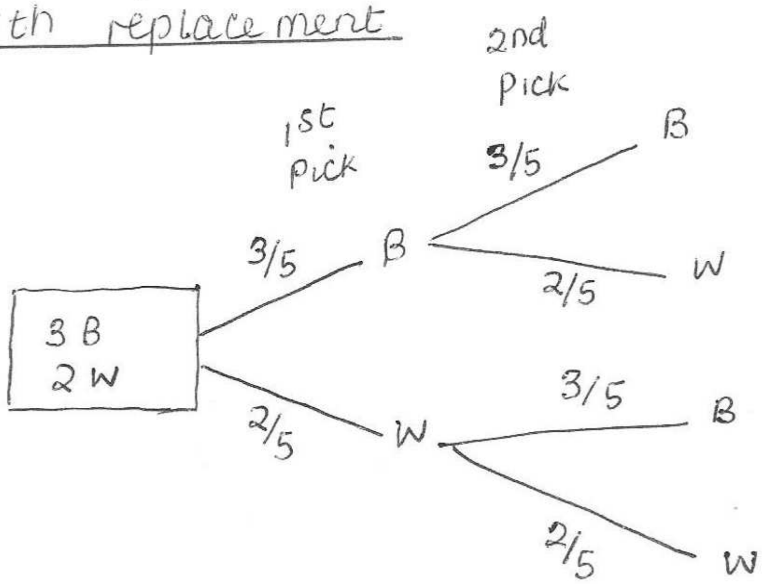
$= \frac{6}{20} + \frac{6}{20}$

$= \frac{12}{20}$

$= \frac{3}{5}$

We can also find these probabilities using a tree diagram.

If we repeat the example 2
(i) <u>with replacement</u>



On the tree diagram, the first time we pick we either pick a black ball or a white ball.
On the second pick, If we picked a black ball the first time, we will either pick a black ball again or a white ball the first time
OR If we picked a white ball the first time, we will pick a black ball or a white ball again.

Therefore $P(B \cap B) = \frac{3}{5} \times \frac{3}{5}$

$= \frac{9}{25}$

(ii) $P(W \cap W) = \frac{2}{5} \times \frac{2}{5}$

$= \frac{4}{25}$

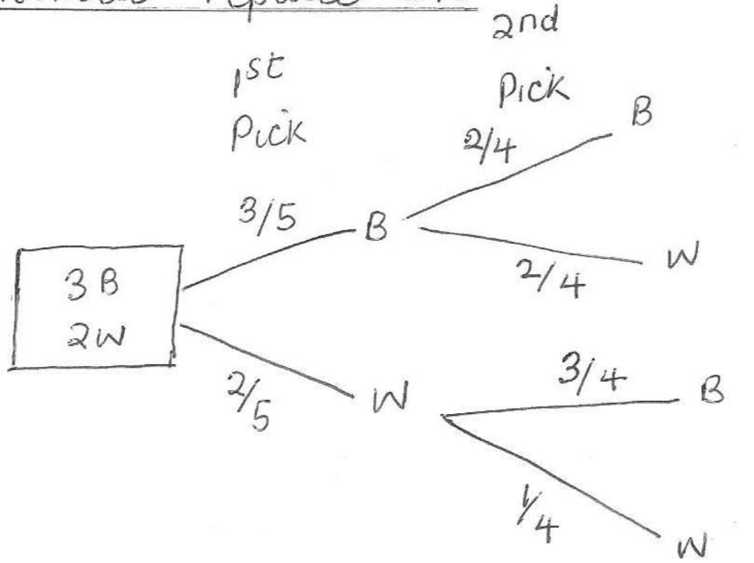
$P(B \cap W) + P(W \cap B)$

$= \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{3}{5}$

$= \frac{6}{25} + \frac{6}{25}$

$= \frac{12}{25}$

(ii) <u>Without replacement</u>



The first time we pick we either pick a black ball or a white ball. If we don't put the ball back, the total number of balls on the second pick will be 4.

Therefore (i) $P(B \cap B) = \frac{3}{5} \times \frac{2}{4}$

$= 6/20$

$= 3/10$

(ii) $P(W \cap W) = \frac{2}{5} \times \frac{1}{4}$

$= \frac{2}{20}$

$= 1/10$

(iii) P(Black and White)

$P(B \cap W) + P(W \cap B)$

$$= \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{3}{4}$$

$$= \frac{6}{20} + \frac{6}{20}$$

$$= \frac{12}{20}$$

$$= \frac{3}{5}$$

Exercise

1. A box contains 5 blue books and 3 green books. 2 books are picked at random one after the other without replacement. Using a tree diagram, find the probability that
   (i) both books are green
   (ii) both books are of the same colour
   (iii) the books are of different colours

2. A bag contains 4 blue pens, 5 red pens and 3 green pens. 2 pens are picked at random from the bag without replacement. Using a tree diagram determine the probability that
   (i) the pens are of the same colour
   (ii) The pens are of different colours.

RANDOM SELECTION

Four identical blue pens were put in a box. 2 pens were picked randomly without replacement.

a) Write the possibility space

b) what is the probability of picking the second blue pen and the third blue pen?

a) In this case we label the blue pens as the first blue pen B_1, second blue pen B_2 and so on.
We then work out the possibility space as follows;
If we pick the first blue pen, we could then either pick the second blue pen or the third blue pen or the fourth blue pen. OR If you pick the second blue pen first, you could then pick the first blue pen or the third blue pen or the fourth blue pen and so on.
The possibility space would then look like below

		2nd Pick			
		B_1	B_2	B_3	B_4
1st Pick	B_1	B_1B_1	B_1B_2	B_1B_3	B_1B_4
	B_2	B_2B_1	B_2B_2	B_2B_3	B_2B_4
	B_3	B_3B_1	B_3B_2	B_3B_3	B_3B_4
	B_4	B_4B_1	B_4B_2	B_4B_3	B_4B_4

NB If we are picking the pens without replacement we cannot pick the first blue pen and pick it again. So B_1B_1, B_2B_2, B_3B_3 and B_4B_4 are not possible.
There fore we have 12 possibilities.

b) The probability of picking the second blue pen and the third blue pen.

We have 2 possibilities; we could either pick the second blue pen first then the third blue pen or we could pick the third blue pen first then the second blue pen. That is B_2B_3 and B_3B_2

~~$\therefore P(B_2B_3) =$~~

$$P(2^{nd} \text{ blue pen and the } 3^{rd} \text{ blue pen}) = \frac{2}{12} = \frac{1}{6}.$$

Try this

A bag contains 5 similar green balls labelled 1-5. Two balls are picked at random one after the other without replacement.

(i) Write down the possibility space.

(ii) Find the probability of picking the third green ball and the fifth green ball.

(iii) What is the probability of having the sum of the numbers on the balls as a prime number?